

Section-A

Note: Each Question has four possible options. Tick (✓) the correct option.

(20x1=20)

1	$\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) =$							
	A	0	B	∞	C	1	D	-1
2	$\lim_{x \rightarrow 0} \left(\frac{\sin ax}{\sin bx} \right) =$							
	A	$\frac{a}{b}$	B	$\frac{b}{a}$	C	$\frac{1}{ab}$	D	ab
3	Which of the following represent $f^{-1}(5)$ if $f(x) = x^{\frac{1}{3}} + 2$							
	A	1	B	3	C	9	D	27
4	if $y = \cos(ax + b)$ then $y_2 = ?$							
	A	$-a \sin(ax + b)$	B	$-a^2 \cos(ax + b)$	C	$a^3 \sin(ax + b)$	D	$-a^2 \sin(ax + b)$
5	A differentiable function $f(x)$ has relative maxima at c if							
	A	$f''(c) = 0$	B	$f''(c) > 0$	C	$f''(c) < 0$	D	$f''(c) = 1$
6	If $y = \sin^{-1} \frac{x}{a}$, then y_1 is							
	A	$(a^2 + x^2)^{1/2}$	B	$(a^2 - x^2)^{-1/2}$	C	$(a^2 + x^2)^{-1/2}$	D	$(a^2 - x^2)^{1/2}$
7	$\int \tan x \, dx =$							
	A	$\ln \cos x + c$	B	$-\ln \sin x + c$	C	$\ln \sec x + c$	D	$\ln \sin x + c$
8	A particle is moving in a straight line and its acceleration is $a = 2t - 7$, then its velocity v is:							
	A	2	B	-5	C	$t^2 - 7 + c$	D	$t^2 - 7t + c$
9	$\int e^x \left[\frac{1}{x} + \ln x \right] dx =$							
	A	$\frac{e^x}{x} + c$	B	$e^x \ln x + c$	C	$\frac{e^x}{\ln x} + c$	D	$xe^x + c$
10	$\int \frac{x}{x+2} dx =$							
	A	$x - \ln(x+2) + c$	B	$\ln(x+2) + c$	C	$\frac{1}{2} \ln(x+2) + c$	D	$x - 2 \ln(x+2) + c$
11	The location in the plane of the point $P(x, y)$ for which $y = 0$ is							
	A	First Quadrant	B	Second Quadrant	C	X-axis	D	Y-axis

12	The equation of line through $(8, -3)$ having slope 0 is							
	A	$y - 3 = 0$	B	$y + 3 = 0$	C	$x - 3 = 0$	D	$x + 3 = 0$
13	The pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ are real and coincident if							
	A	$h^2 = ab$	B	$h^2 < ab$	C	$h^2 > ab$	D	$h^2 + ab = 0$
14	The inequality $2x - 3y \leq 6$ has a solution							
	A	$(4, 2)$	B	$(2, -3)$	C	$(0, -3)$	D	$(4, 0)$
15	The equation $x^2 + y^2 + 4x + 12y + 15 = 0$, represents							
	A	Circle	B	Parabola	C	Ellipse	D	Hyperbola
16	The directrix of the parabola $x^2 = -16y$, is							
	A	$x + 4 = 0$	B	$x - 4 = 0$	C	$y + 4 = 0$	D	$y - 4 = 0$
17	In an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the length of Latusrectum is							
	A	$\frac{2a}{b^2}$	B	$\frac{2b^2}{a}$	C	$\frac{2b}{a^2}$	D	$\frac{2b^2}{a^2}$
18	Which of the following triples can be direction angles of a single vector							
	A	$45^\circ, 45^\circ, 60^\circ$	B	$30^\circ, 45^\circ, 60^\circ$	C	$45^\circ, 60^\circ, 60^\circ$	D	$30^\circ, 45^\circ, 90^\circ$
19	The projection of $\hat{i} + \hat{j}$ along \hat{k} is							
	A	1	B	0	C	$\frac{1}{\sqrt{2}}$	D	$-\frac{1}{\sqrt{2}}$
20	The value of $2\hat{i} \times 2\hat{j} \cdot \hat{k}$ is							
	A	0	B	1	C	2	D	4

Section-B

Note: Attempt twelve (12) short questions.

Q. 1 Write short answers of any eight parts.

(12x4=48)

(i) If $f(x) = \frac{1}{\sqrt{x-1}}$ and $g(x) = (x^2 + 1)^2$ find (a) $f \circ f(x)$ (b) $g \circ f(x)$

(ii) evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$

(iii) Differentiate $\frac{(1 + \sqrt{x}) \left(x - x^{\frac{3}{2}} \right)}{\sqrt{x}}$ w.r.t x

(iv) If $y = \tan \left[2 \tan^{-1} \frac{x}{2} \right]$ then find $\frac{dy}{dx}$.

(v) Show that $2^{x+h} = 2^x \left\{ 1 + (\ln 2)h + \frac{(\ln 2)^2 h^2}{2!} + \frac{(\ln 2)^3 h^3}{3!} + \dots \right\}$

(vi) Solve the differential Equation $y - x \frac{dy}{dx} = 2(y^2 + \frac{dy}{dx})$.

(vii) Evaluate the integral $\int \frac{dx}{\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x}$

(viii) Evaluate the integral $\int_{-1}^5 |x - 3| dx$

(ix) The point A (-1, 2), B (6,3), C (2,-4) are vertices of a triangle. Show that the line joining the mid-point D of AB and the midpoint E of AC is parallel to BC and $DE = \frac{1}{2} BC$.

(x) Find the distance of point p (6, -1) to the line $6x - 4y + 9 = 0$.

(xi) Graph the feasible regions subjects to following constraints $2x-3y \leq 6$, $2x+y \geq 2$, $x \geq 0$, $y \geq 0$.

(xii) Find equation of circle passing through the A(3, -1), B(0,1) and having a centre at $4x-3y-3=0$.

(xiii) find equation of parabola having focus (-3, 1) and directrix $x=3$.

(xiv) find equation of ellipse having focus (0, 3), centre (0,0) and vertex at (0,4).

(xv) Find constant α so that $\vec{v} = 3\mathbf{i} + \alpha\mathbf{j} + 4\mathbf{k}$ and $\vec{w} = 4\mathbf{i} + 5\mathbf{j} + \alpha\mathbf{k}$ are perpendicular.

(xvi) Find volume of tetrahedron whose vertices are are A (-2, 1, 4), B (3, 2, 5), C (-3, -5, 0), D (5, 8, 9)

Section-C

Note: Attempt any four questions from this section.

(4x8=32)

Q. 3 Find values of m and n if given function is continuous at $x=3$, also state domain and range of function

taking $m=1$ and $n=2$ $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x+9 & \text{if } x > 3 \end{cases}$

Q. 4 From ab-initio method, find derivative $\frac{1}{(ax+b)^n}$

Q. 5 Evaluate the integral $\int \frac{2x^2-1}{x^4+x^2+1} dx$.

Q. 6 (a) Find the coordinates of the vertices of triangle formed by the lines $x - 2y - 6 = 0$, $3x - y + 3 = 0$; $2x + y - 4 = 0$

(b) Also find measure of angles of triangle.

Q. 7 A farmer plans to mix two types of food to make a low cost feed for animals. A bag of food P cost him Rs. 40 and contains 5 units of proteins and 4 units of vitamins. A bag of food Q cost him Rs. 50 and contains 4 units of proteins and 8 units of vitamins. How many bags of food P and Q should be consumed in order to have 120 units of proteins and 144 units of vitamins at minimum costs?

Q.8 By rotation of axes, eliminate the xy -term in the equation $9x^2+12xy+4y^2+2x-3y=0$. Identify the conics and find its elements.